ASSIST *recommends...*

Evaluating Light Source Flicker for Stroboscopic Effects and General Acceptability

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Abstract

This issue of *ASSIST recommends* describes an objective, calculation-based method for predicting the perception of stroboscopic effects due to the interaction of motion and flickering light, herein called the stroboscopic acceptability metric (SAM). A perceptual flicker-manifested spatial contrast metric (C_P) is defined which is analogous to the perceptual temporal modulation metric (M_P) for the direct detection of temporal flicker (see *ASSIST recommends Volume 11, Issue 3: Recommended Metric for Assessing the Direct Perception of Light Source Flicker*). Furthermore, the C_P metric is used to extend empirical results on the general acceptability of light sources for minimizing stroboscopic effects (see *ASSIST recommends Volume 11, Issue 1: Flicker Parameters for Reducing Stroboscopic Effects from Solid-state Lighting Systems*) to any waveform type.

Introduction

As opposed to directly perceived flicker, stroboscopic effects are not revealed by the detection of temporally fluctuating light signals, but rather by the conversion of temporal fluctuations into spatial patterns. Therefore, the perception of stroboscopic flicker depends on the detection of spatial contrast, which is a fundamentally different visual perception than temporal flicker. Consequently, the visual characterization of stroboscopic flicker must ultimately involve spatial contrast.

In the course of evaluating light sources for potential stroboscopic phenomena, one must realize that spatial contrast produced by a flickering light source cannot be assessed by consideration of only the temporal waveform of the light source itself. Rather, one must consider, in addition to the light source, its movement and size (i.e. visual angle) or the movement, size, and reflectance of objects that it illuminates if not directly viewed. In this regard, stroboscopic flicker is analogous to color rendering; evaluation of color rendering requires that an object or objects with particular spectral reflectance be specified in addition to the spectral power distribution of the light source itself. Likewise, for evaluating stroboscopic effects a moving stimulus must be defined in addition to the temporal waveform of the light source.

A methodology for quantifying the detection of stroboscopic effects from modulated light sources is described in the section on modeling (Appendix A). First, equations for calculating the resulting spatial contrast produced by the interaction of stimulus movement and temporal light modulation are derived. This is purely a physical description of the stimulus presented to the observer, which has been mostly ignored in the literature on flicker and transient light artifacts (TLA). After the physical stimulus is defined, a linear systems approach is used to apply the human sensitivity for detecting spatial contrast to the physical spatial contrast presented to the observer in order to determine the visibility of stroboscopic phenomena.

In order to evaluate light sources for potential stroboscopic flicker, viewing conditions need to be defined that specify the size and speed of the luminous objects involved. Fortunately, there is an optimum object size and optimum speeds for detecting stroboscopic effects. A perceptual contrast metric, C_P, is defined using these optimum speeds and size. The ratio of C_P of a light source waveform over C_P of a reference waveform enables different light sources to be compared based on their potential for exhibiting stroboscopic effects. Lastly, C_P ratios can be used to extend the limited empirical data on light source waveform acceptability to all other waveforms, resulting in an acceptability metric for light sources regarding their potential for stroboscopic flicker.

The sections below describe the stroboscopic acceptability metric (SAM) and the procedure for calculating C_P and C_P ratio. For details on the method used to model stroboscopic flicker, see Appendix A. For details on the method for computing optimum flicker-manifested contrast, C_P, see Appendix B.

Stroboscopic Acceptability Metric (SAM)

The acceptability of perceived stroboscopic effects produced under square-wave modulation of varying modulation depths (% flicker) was empirically modeled by Bullough et al.(2012) and ASSIST (2012). Acceptability, *a*, is given by:

$$a = 2 - \frac{4}{1 + \frac{f}{130log_{10}(p) - 73}}$$

Where *f* is the dominant or fundamental frequency of modulation and *p* is percent flicker. The numerical values of *a* are interpreted as follows:

+2 very acceptable
+1 somewhat acceptable
0 neither acceptable nor unacceptable
-1 somewhat unacceptable
-2 very unacceptable

DUT in the above equation stands for device under test, that is, the light source being evaluated. Presumably, subjects' acceptability responses are related to how easily flicker-manifested contrast is detected; the higher the detection probability the lower the acceptability. This is roughly supported by the contour plots of detection and acceptability versus frequency and percent flicker of Bullough et al. (2012) The detection of flicker-manifested contrast is known to depend on the shape of the light waveform, so it is likely that acceptability also depends on wave shape. A method for extending the acceptability formula, generated using square-wave modulation, to all possible light waveform types is to scale percent flicker, *p*, by the ratio of perceptible flicker contrast for the waveform in question to that for a square wave, (C_P test waveform)/(C_P square wave). Essentially, what originally was percent flicker that merely describes the amplitude variation of the waveform is now replaced by a measure of perceptual flicker that accounts for wave shape and which is normalized to square-wave equivalent values for use in the Bullough et al. formula.

$$a = 2 - \frac{4}{1 + \frac{f}{130log_{10}(p_P) - 73}}$$

where

$$p_P = p\frac{C_P(DUT)}{C_P(squarewave, f_{DUT}, p_{DUT})}$$

The calculation of C_P requires a relative light output waveform as a function of time as input. The calculation uses an optimal object profile and speed for revealing stroboscopic flicker. C_P is independent of the frequency of the light waveform unless an upper limit is placed on the speed of objects. No upper speed limit is used for C_P because the formula for acceptability already includes a dependency on frequency which was determined by the conditions used in the studies leading to the acceptability ratings. The ratio of C_P values, therefore, is only dependent on the light waveform and, for non-symmetrical waveforms, percent flicker. C_P values do not vary with percent flicker for waveforms that are symmetrical about their mean value (e.g., sine, square, ramp, sawtooth). A simple table of *p* multipliers, named C_P ratio values, can be generated for different waveforms and percent flicker amounts. Table 1 lists C_P ratios for several common wave shapes.

Table 1. C_P ratios for several common wave shapes.

Wave shape	**C_P ratio (DUT/square)**
	% Flicker*: Threshold, 10%, 50%
Square	1.00
Sine	0.78
Rectified sine	0.66, 0.65, 0.59
Ramp	0.64
Rectangular 20% duty cycle	0.59, 0.63, 0.84
Rectangular 80% duty cycle	0.59, 0.55, 0.45
Sawtooth	0.50
Rectangular 10% duty cycle	0.31, 0.34, 0.51

* For non-symmetrical waveforms, the ratio dc(DUT)/dc(square) changes with % flicker which then affects C_P ratio..

Example: Calculate an acceptability rating for a 400 Hz rectified sinewave at 20% modulation.

Interpolating a value from the above table, the C_P ratio of a rectified sinewave of 20% flicker is approximately 0.63. Multiplying this by the rectified sinewave modulation gives the perceptual modulation for this waveform.

$$p_P = p \times (C_P ratio) \approx 20 \times 0.63 = 12.6$$

$$a = 2 - \frac{4}{1 + \frac{f}{130 log_{10}(p_P) - 73}} = 2 - \frac{4}{1 + \frac{400}{130 log_{10}(12.6) - 73}} = 1.404$$

For comparison, a 400 Hz square wave at 20% modulation gives *a* = 1.225.

Rectangular waveforms other than square waves (50% duty cycle) are non-symmetrical with respect to their mean (dc) value. This non-symmetry causes the dc value to vary with changes in percent flicker and this affects the spectral contrast values according to equation 7 (Appendix A). Compared to a symmetrical square wave, spatial contrast increases for decreasing duty cycle and decreases for duty cycles less than 50% when percent flicker is held constant. Therefore, the C_P ratio changes for different amounts of percent flicker for these non-symmetrical waveforms. Figure 1 plots the C_P ratio as a function of duty cycle for different amounts of percent flicker. For small amounts of percent flicker there is little change in C_P ratio; however, for large amounts of flicker C_P ratios increase dramatically for short duty cycles. This behavior is consistent with observations that stroboscopic effects are most pronounced under short duty cycle, high modulation sources. Strobe lights, with their near 100% flicker and short flashes, exemplify this.

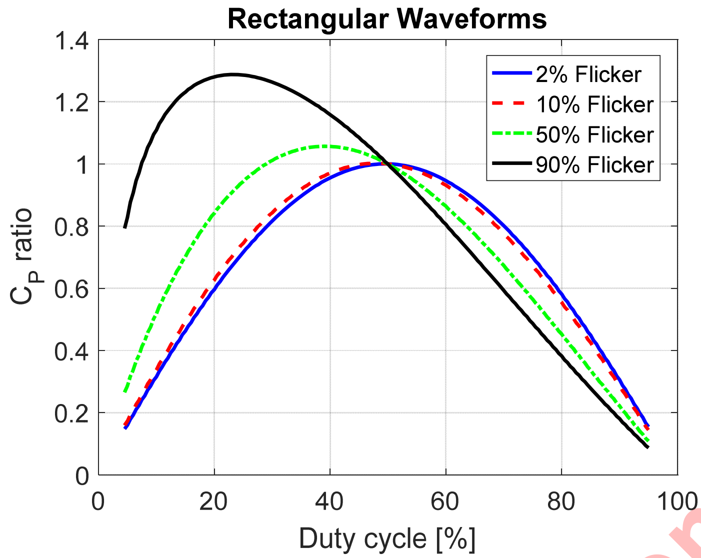

Figure 1. C_P ratio for rectangular waveforms as a function of duty cycle for several amounts of percent flicker.

Step-By-Step Procedure for Calculating C_P

The following is a list of step-by-step instructions for calculating C_P for an arbitrary light source with Matlab-styled pseudo code expressions. Certain steps require knowledge of photometry and digital signal processing that are not covered here, but are basic to their respective fields. For insights into the signal processing of the light output waveforms, example Matlab scripts and functions are available that calculate C_P (see http://www.lrc.rpi.edu/programs/solidstate/assist/recommends/flicker.asp).

1. Measure the relative light output of the device under test (DUT) as a function of time, ϕ(t).
2. Specify or measure the reflectance (relative luminance) profile of the moving object (light source). Both reflectance and relative luminance should be in the range from 0 to 1.
3. Convert the time domain of the light vector to visual angle domain by multiplying the time argument by the velocity of the movement and dividing by the viewing distance.
   - $\phi(t) \Rightarrow \phi(tv) = \phi(x) \Rightarrow \phi\left({}^{x}/_{d}\right) = \phi(\theta_V)$
4. If not already expressed as a function of visual angle, convert the object profile domain to visual angle by dividing object dimensions by the viewing distance.
   - $o(x) \Rightarrow o\left({}^{x}/_{d}\right) = o(\theta_V)$
5. Apply a persistence-of-vision time window to the object profile. This window is modeled as a decaying exponential with a time constant of 45 milliseconds. Velocity, v, is angular velocity in units of radians/s, and θ_V is visual angle in radians.

- $o(\theta_V) = o(\theta_V).* \exp\left(-\frac{1}{0.045} * \theta_V/v\right)$

6. Apply window function to the light vector to minimize finite sampling effects. A Hann window works well.
   - Window = window(@hann,length(ϕ(θ_V)));
   - $\phi_{Window}(\theta_V)$ = Window.* ϕ(θ_V);
7. Compute Fourier transform of light and object vectors.
   - $o(\theta_V) \overset{\mathcal{F}}{\Leftrightarrow} O(v)$ and $\phi(t) \overset{\mathcal{F}}{\Leftrightarrow} \Phi(\omega)$
8. Multiply the Fourier transformed light and object vectors together, element-by-element, convert to magnitude, and then divide each element by the dc value. The result is the physical contrast spectrum.
   - $C(\omega) = \frac{|\Phi(j\omega)O(j\omega)|}{|\Phi(0)O(0)|}$
9. Multiply physical contrast spectrum by the CSF function element-by-element. Interpolation of CSF values is likely necessary.
   - $C_P(\omega) = C(\omega)CSF(\omega)$
10. Take the Euclidean norm of the C_P vector to arrive at the overall C_P value

$$C_P = |C_P(\omega)| \cong \sqrt[2]{\sum_{i \neq 0} (C_{Pi})^2} \begin{cases} < 1 & \text{not visible} \\ = 1 & \text{just visible} \\ > & \text{visible} \end{cases}$$

Step-By-Step Procedure for Calculating C_P Ratio

When calculating C_P ratios, three simplifications can be made. First, the contrast sensitivity function (CSF) appears as a factor in both the numerator and denominator, and so it cancels. Second, the object profile waveform can be arbitrarily chosen, so choosing a sine waveform reduces its frequency spectra to a single component scalar value. Third, the persistence of vision window is not necessary because its effects are canceled by it appearing in both the numerator and denominator. Below is a step-by-step procedure for calculating a C_P ratio with Matlab-styled pseudo code expressions. An example Matlab function is available that calculates C_P ratio (see http://www.lrc.rpi.edu/programs/solidstate/assist/recommends/flicker.asp).

1. Measure the relative light output of the device under test (DUT) as a function of time, ϕ(t).

2. Calculate percent flicker and dc level.
   - p = 100*(max(ϕ(t))-min(ϕ(t)))/(max(ϕ(t))+min(ϕ(t)))
   - dc = mean(ϕ(t))
3. Determine the frequency of the maximum spectral component (usually the fundamental frequency).
   - Compute Fourier transform of light output waveform: $\phi(t) \overset{\mathcal{F}}{\Leftrightarrow} \Phi(\omega)$
   - f_{max} = argmax(Φ(ω))
4. Compute time series vector of square waveform values with a fundamental frequency equal to f_{max}..
   - ϕ_{SqWave}(t) = (A*square(2*pi* f_{max} *t,dutyCycle))+dc, where square is a Matlab function for producing square waves, A = dc*p/100, and dutyCycle = 50
5. Apply window function to the DUT and square wave vectors to minimize finite sampling effects. A Hann window works well.
   - Window = window(@hann,length(ϕ(t)));
   - ϕ_{Window}(t) = Window.* ϕ(t);
   - $\phi_{SqWaveWindow}$(t) = Window.* ϕ_{SqWave}(t);
6. Compute single component discrete Fourier transform (DFT) for both DUT and Square wave vectors.
   - A_DUT = sum(ϕ_{Window}(t).*exp(1i* f_{max} *2*pi*t));
   - A_sqWave = sum($\phi_{SqWaveWindow}$(t).*exp(1i* f_{max} *2*pi*t));
7. Divide A_DUT by A_SqWave to compute C_P ratio.
   - C_P Ratio = A_DUT/A_SqWave

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Alliance for Solid-State Illumination Systems and Technologies (ASSIST). 2012. *ASSIST recommends: Flicker Parameters for Reducing Stroboscopic Effects from Solid-state Lighting Systems*. Vol. 11, Iss. 1. Troy, N.Y.: Lighting Research Center. Internet: http://www.lrc.rpi.edu/programs/solidstate/assist/recommends/flicker.asp.

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About ASSIST

The Alliance for Solid-State Illumination Systems and Technologies (ASSIST) was established in 2002 by the Lighting Research Center as a collaboration among researchers, manufacturers, and government organizations. ASSIST's mission is to enable the broad adoption of solid-state lighting by providing factual information based on applied research and by visualizing future applications.

Appendix A – Modeling Stroboscopic Flicker

The movement of a temporally varying luminous object within the field of view of an imaging device results in a transformation of temporal variation to spatial variation. This transformation is analogous to the way rapidly fluctuating electrical events are revealed as easily seen waveforms on an oscilloscope screen. To start the analysis, consider Figure A1 which depicts a one-dimensional reflective line moving horizontally across a dark background with a velocity **v** (e.g., 4 m/s). The line is illuminated by a modulated light source having frequency f_{Light} (e.g., 100 Hz), and viewed by an observer at a distance, d (e.g., 4 m). To the observer the light source appears to have a steady light output because 100 Hz is above the critical flicker fusion frequency. As the line moves across the observer's field of view, however, the perception is not of a steadily moving object, but rather a series of bright lines fixed in space. The spacing of the lines is given by the product of the velocity and the frequency of the light modulation (velocity × (1/time) = displacement). In terms of visual angle, the small angular approximation, tangent(θ) ≈ θ is used to arrive at a simple expression for the spatial frequency, $f_{Spatial}$, of the resulting stimulus.

$$f_{spatial} = \frac{f_{Light} d}{v}$$

Equation 1

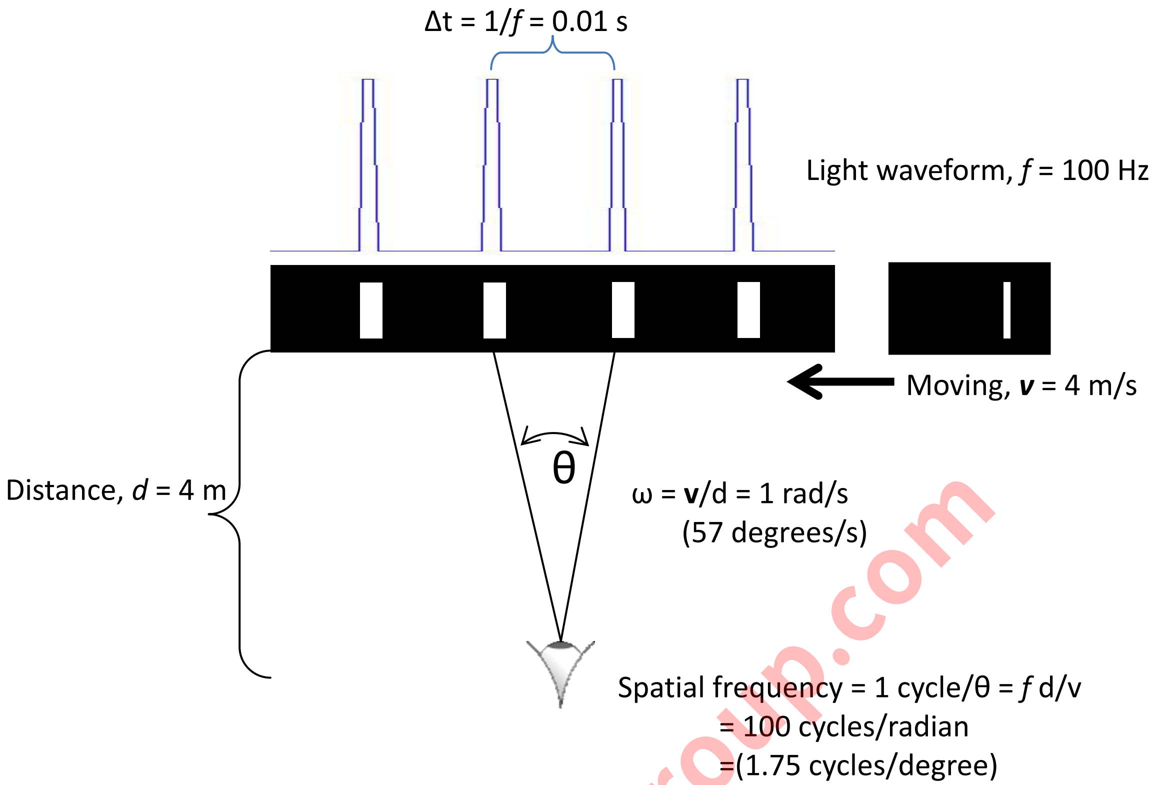

Figure A1. Schematic diagram depicting how a moving object is revealed to an observer as a spatial contrast pattern when viewed under a modulated light source. The production of a spatial contrast pattern in this manner is called a stroboscopic effect.

The simple analysis leading to equation 1 only provides the frequency of the spatial pattern. To more fully describe the spatial irradiance pattern produced on an imaging detector, such as the human eye, it is necessary to consider the waveform shape of the temporally modulated light, as well as the shape profile of the reflectance of the moving object. Also, for imaging systems it is more convenient to express dimensions and velocities in terms of visual angle rather than actual distances because visual angle ultimately determines visibility. Physical modeling of the stroboscopic effect has been done previously by Ku et al. (2015) who derived integral equations to model the superposition of reflected light over the exposure duration. While the end result is the same, the analysis here follows a linear systems approach that facilitates computations of complex stimuli and, as described later, enables the perceptual response of the human visual system to be applied as a frequency dependent filter.

The light waveform shape is a function of time, $\phi(t)$, and the object can be described by its reflectance profile as a function of position, *o(x)*. As shown in Figure A1, movement of the object reveals the temporal variation of light flux as a spatial variation. We can account for this mathematically by multiplying the time dependency by velocity, thereby converting the dependency of flux on time to a variation of flux with distance and ultimately visual angle. Similarly, for the object reflectance profile, which is already a function of distance, we can express the object in terms of visual angle. These conversions to visual angle are expressed in equations 2a and 2b.

Light waveform: $\phi(t) \Rightarrow \phi(tv) = \phi(x) \Rightarrow \phi\left({}^{x}/_{d}\right) = \phi(\theta_V)$

Object profile: $o(x) \Rightarrow o\left({}^{x}/_{d}\right) = o(\theta_V)$

Equation 2a, 2b

With these conversions, both the object profile and the light variation are expressed as functions of visual angle. The resulting contrast pattern presented to the observer is the superposition of the product of illumination and reflectance at all locations over the time interval during which the perceptual image formed, taking into account that the reflective object is moving. This superposition of light and object reflectance is accomplished mathematically by a convolution. A convolution is a mathematical operation pictured as the result of sliding one function over another while integrating the product of the two over all points. The resulting profile takes on characteristics of both functions. Equation 3 shows the convolution of the light waveform with the object reflectance profile resulting in a physical variation of light intensity with visual angle, i.e. a visual stimulus.

$$\text{Physical stimulus}(\theta_V) = (\phi * o)(\theta_V)$$

Equation 3

Equation 3 is a one-dimensional spatial description of the contrast pattern presented to an observer. When velocity is constant, this one dimensional analysis is sufficient for characterizing the two-dimensional visual stimulus by constructing it from multiple parallel line profiles. Only in cases of significant acceleration (linear or centripetal) would a more complicated model be needed. Using a linear systems approach, the perceived visual stimulus is modeled by convolution of the physical stimulus with the spatial impulse response of the visual system. Also, the time response of the visual system needs to be considered as it relates to the persistence of vision that enables the superposition of flux to buildup and form an image. The persistence of vision is simply modeled as a window in time over which the convolution integral described by equation 4 is evaluated.

$$\text{Perceptual Stimulus} = ([(\phi * o')(\theta_V)] * \{\text{Perceptual Impulse Response}\})(\theta_V)$$

Equation 4

The time window is applied by point-by-point multiplication of the object profile, $o(\theta_V)$, by the window profile. A simple exponential decay window with a time constant of 45 milliseconds is currently used as given by equation 5.

$$o'(\theta_V) = o(\theta_V)\exp\left(\frac{1}{0.045}\frac{\theta_V}{v}\right),$$

Equation 5

θ_V/v is a time quantity (seconds) when the speed, v, is in units of radians/s and θ_V is in units of radians.

Unfortunately, the complete perceptual impulse response of the visual system is not known. However partial information of the perceptual impulse function is well known as the contrast sensitivity function (CSF) and it is readily measured experimentally (Olzak and Thomas 1985). The CSF is the magnitude response of the visual system to luminous spatial contrast as a function of frequency. In preparation for using the CSF, equation 4 is transformed to the spatial frequency domain via a Fourier transform leading to equation 6.

$$\text{Perceptual Stimulus}(j\omega) = (\Phi(j\omega)O(j\omega))\{\text{Perceptual Impulse Response}\}(j\omega)$$

Equation 6

The CSF is measured and applied in terms of contrast, so the physical stimulus must also be expressed in terms of contrast. A Weber contrast is obtained by dividing the physical stimulus by the average stimulus intensity which is equivalent to dividing by the zero-frequency component (dc component) as written in equation 7.

$$C(\omega) = \frac{|\Phi(j\omega)O(j\omega)|}{|\Phi(0)O(0)|}$$

Equation 7

Finally, the magnitude of the perceptual stimulus (ignoring phase) is found by expressing equation 6 in terms of contrast as defined by equation 7. The result is equation 8 where C_P is the perceptual contrast.

$$\text{Perceptual Stimulus}(\omega) = C(\omega)CSF(\omega)$$
$$C_P(\omega) = C(\omega)CSF(\omega)$$

Equation 8

Human factors studies at the Lighting Research Center have shown that the visual system approximates a linear system where the total spectral power over all spatial frequencies of the stimulus is given by the quadrature sum of spectral components. Furthermore, when the CSF is provided in absolute units of 1/(contrast needed for detection) then a measure of perceptual stimulus strength is given by equation 9.

$$C_P = |C_P(\omega)| \cong \sqrt[2]{\sum_{i \neq 0} (C_{Pi})^2} \quad \begin{cases} < 1 & \text{not visible} \\ = 1 & \text{just visible} \\ > & \text{visible} \end{cases}$$

Equation 9

A value of 1 for equation 9 corresponds to a stimulus at the threshold of detection while values less than 1 are not detectable and values greater than 1 are detectable with increasing probability as the value increases.

Appendix B – Computing Optimum Flicker-manifested Contrast, C_P

Flicker-manifested contrast depends on the object being viewed (size, speed, reflectance), but optimal values of object size, speed and reflectance exist that maximize the C_P. Clearly, the optimal reflectance variation across the object would be 1, that is, a 100% reflective object moving across a perfectly black background.

The optimal width, w_O, of the object stimulus is given by the peak of the CSF function.

$$w_O = \frac{1}{2f_{max}} = \frac{1}{2\times 200} = 0.0025 \ \ [\text{radians}],$$

Equation 10

where f_{max} is the frequency of maximum contrast sensitivity for humans, approximately 200 cycles/radian (3.5 cycles/degree). w_O is approximately 0.0025 radians. At 1 meter viewing distance this corresponds to a 2.5 mm thick stick. The optimum object profile, $o(\theta_V)$ is a repeating pattern of white ($\rho = 1$) and black ($\rho = 0$) bars of width w_O.

The optimum speed of the object, v_O, for producing stroboscopic spatial contrast depends on both the object width and the frequency of the modulated light.

$$v_O = \frac{2w_O\omega_{max}}{2\pi} = \frac{w_O\omega_{max}}{\pi} \quad \left[\frac{\text{radians}}{\text{s}}\right],$$

Equation 11

where ω_{max} is the frequency of highest modulation of the light waveform ($\omega_{max} = \text{argmax}(\Phi(\omega))$ [radians/s], where $\Phi(\omega)$ is the Fourier transform of $\phi(t)$).

Typically, ω_{max} is the fundamental frequency, but not necessarily so. For example, v_O = 0.5 m/s for a 100 Hz waveform and an object at 1 meter distance. With the optimum object and speed defined, C_P is calculated according to equation 9. Equations 12a and 12b show the calculation equations for C_P in integral and discrete forms, respectively.

$$C_P = \sqrt[2]{\int (|\Phi(\nu)O(\nu)|CSF(\nu))^2 d\nu}$$

Equation 12a

$$C_P = \sqrt[2]{\sum_i |(\Phi_i O_i|CSF_i)^2}$$

Equation 12b

The frequency domain transforms of the light waveform and object profile are given below.

$\Phi(\nu) \equiv$ Fourier transform of light waveform where the time argument of the light waveform has been converted to spatial frequency [radians/radian] by dividing the frequency, ω [radians/s], by the velocity of the object [radians/s], $\nu = \omega/v_O$ [unitless].

$$\phi(t) \overset{\mathcal{F}}{\Leftrightarrow} \Phi(\omega) \rightarrow \Phi(\nu)$$

Equation 13

$O(\nu) \equiv$ Fourier transform of object reflectance profile. The time response of the eye (a.k.a. persistence of vision) is included in the object profile by weighting the object profile with the time response of the eye given by equation 5 and repeated in equation 14 for clarity. If the object profile is in units of visual angle [radians] the argument of the Fourier transform is in units of [radians/radian] (unitless). The object profile is the reflectance of the object (and its background) as a function of visual angle, θ_V, ranging from 0 to 1 and it is also unitless. For the general metric this profile is given by w_O.

$$o'(\theta_V) = o(\theta_V)\exp\left(\frac{1}{0.045}\frac{\theta_V}{v}\right)$$

Equation 14

$$o'(\theta_V) \overset{\mathcal{F}}{\Leftrightarrow} O(\nu)$$

Equation 15

$CSF(\nu) \equiv$ Human contrast sensitivity function in absolute units (not relative). This is equivalent to 1/(modulation threshold). It is also a unitless quantity [1/contrast].

The average CSF for 8 people measured at the Lighting Research Center for viewing stroboscopic contrast patterns produced by a rotating sector disk is shown in Figure B1. Tabulated values are given in Table B1. These tabulated values can be interpolated to match the spatial frequencies of the object and light waveforms.

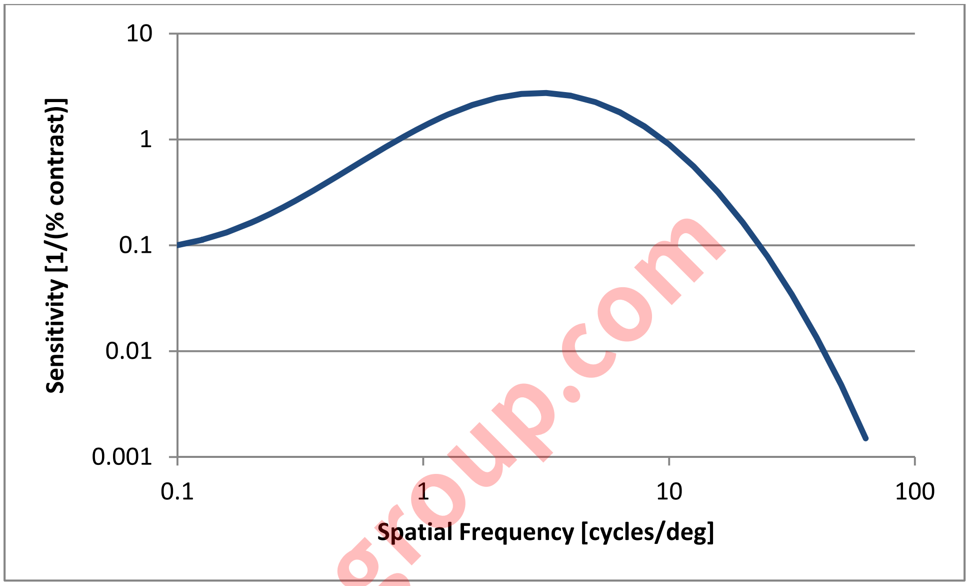

Figure B1. Average CSF for 8 subjects

Table B1. Tabulated values of the average CSF for 8 subjects. (Subject ages 22 to 52, mean 33, 3 female).

cycles/deg	CSF	cycles/deg	CSF	cycles/deg	CSF
0.100	0.101	1.000	1.333	10.00	0.898
0.126	0.113	1.259	1.719	12.59	0.556
0.159	0.133	1.585	2.118	15.85	0.316
0.200	0.164	1.995	2.468	20.0	0.165
0.251	0.210	2.51	2.698	25.1	0.078
0.316	0.280	3.16	2.748	31.6	0.034
0.398	0.382	3.98	2.593	39.8	0.014
0.501	0.528	5.01	2.257	50.1	0.005
0.631	0.730	6.31	1.808	63.1	0.002
0.794	0.998	7.94	1.330		